Ph.D. Programme in Mathematics

Model Question Paper

RESEARCH APTITUDE ASSESSMENT TEST

Time : 2 Hours **Max. Marks : 75**

I. Part A: Multiple Choice Questions 30 x 1 mark = 30 marks

Choose the correct Response viz., A, B, C, D or E for the Questions from 1 - 30 which carry ONE mark each. Please NOTE that an **incorrect response** will attract **negative marking**. (For Multiple Choice question with 5 options, ¼th mark shall be deducted for an incorrect answer.)

1. Consider the following system of equations:

$$\begin{aligned} x_1 + x_3 &= 3 \\ x_1 - x_2 - x_3 &= 1 \\ -x_1 + x_2 &= 4 \end{aligned}$$

( )

The above system of linear equations is:
A) consistent with infinitely many solutions
B) consistent with a unique solution
C) inconsistent
D) inconsistent but has many solutions
E) inconsistent but has a unique solution

2. The eigen values of a skew-symmetric matrix are ( )
A) all zeros
B) always real
C) always purely imaginary
D) always zero and purely imaginary
E) does not always exist

3. The rank of the matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ is ( )
A) 0 B) 1 C) 2 D) 3 E) 4

4. Which of the following is a linearly independent set of vectors over $\mathbb{R}$? ( )
A) {} B) $\{0\}$ C) $\{1,2\}$ D) $\{(1,1), (2,2)\}$ E) $\{(0,0), (1,1)\}$

5. Given that on tossing two fair coins one head appears. What is the probability that head appears on the other coin as well? ( )
A) ½ B) 1/3 C) ¼ D) 1/6 E) cannot be determined

6. If E denotes expectation, the variance of a random variable X is given by ( )
A) $E[X^2] - E^2[X]$ B) $E[X^2] + E^2[X]$ C) $E[X^2]$
D) $E^2[X]$ E) $E[X^2 + E^2[X]]$

7. Which one of the following number systems is algebraically closed? ( )
A) Natural numbers B) Integers C) Real
D) Complex E) Quaternions

8. Which of the fields is a degree 2 extension of $\mathbb{R}$? ( )
A) $\mathbb{Q}(\sqrt{2})$ B) $\mathbb{Q}(i)$ C) $\mathbb{R}$ D) $\mathbb{C}$ E) None of the above

9. Which one of the following maps on the complex plane is NOT analytic? ( )
A) constant B) scaling C) translation
D) rotation E) reflection about real axis

10. What is the order of the Dihedral group D_5? ( )
A) 5 B) 10 C) 20 D) 25 E) None of the above

11. Which of the following spaces is not separable? ( )
A) c B) ℓ^1 C) ℓ^2 D) ℓ^∞ E) None of the above

12. Which of the following is weakly convergent but not strongly? ( )
A) $\{e_n : n \in \mathbb{N}\}$ B) $\{(\xi_n^k) : \xi_n^k = 1,\ n \in \mathbb{N}\}$
C) $\{(\xi_n^k) : \xi_n^k = \xi_n,\ n \in \mathbb{N}\}$ D) $\{(\xi_n^k) : \xi_n^k = \xi_n + \frac{1}{n},\ n \in \mathbb{N}\}$
E) None of the above

13. The residue at the $z = 0$ for the function $f(z) = \frac{1}{z^2+z}$ is given by _____ ( )
A) 1 B) 0 C) πi D) $2\pi i$ E) does not exist

14. Classify the following partial differential equation: ( )
$u_x + u_y + u^2 = f(x)$
A) linear B) semi-linear C) quasi-linear
D) fully nonlinear E) none of the above

15. Which one of the following is true about the solution of the following initial value problem? ( )

$y' = y^{-2}(2-3x)\,, \quad y(0) = 19$:

A) Non-existent B) Trivial C) Infinite

D) Unique E) Vacuous

16. to 30. …

Part - B

II. Answer any 9 of the following in about 150 words each in the sheets provided with the question paper:

(9 x 5 = 45 marks)

1. Show that the transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (0, x, y)$ is not diagonalizable by quoting relevant results.

2. Is the function $f(x) = \begin{cases} x \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$ differentiable at $x = 0$? Justify your answer.

3. Show using induction that $1 + 3 + 5 + \cdots + (2n - 1)$ is a square for $n \in \mathbb{N}$.

4. Determine the radius of convergence of the series $\sum_{k=1}^{\infty} \frac{z^{2k}}{4^k k^k}$.

5. Write the iterative equation to solve the polynomial equation $x^3 + 4x - 9 = 0$ numerically using Newton-Raphson Method.

6. Show that in an inner product space over the reals $\mathbb{R}$, two non-zero vectors are orthogonal if they satisfy the Pythagoras theorem.

7. to 12. …..

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